CURRICULUM GUIDE

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CURRICULUM GUIDE
GRADE VII MATHEMATICS

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PREFACE

This statement has been prepared under the direction of the Junior High School Mathematics Sub-Committee of the Department of Education, Alberta, as a guide for teachers using either of the 1965 authorizations.

Teachers who have had experience with these texts during the experimental period have generously provided time and effort to the preparation of this bulletin. The intention is not to repeat materials included in either the Teacher's Edition or the Teaching Guide, but rather to offer practical suggestions based on the experiences of these teachers.

ACKNOWLEDGEMENT

The Department of Education acknowledges with appreciation the contribution of the following committee members, subcommittee members and teachers of experimental classes in mathematics, to the preparation of this *Curriculum Guide for Grade Seven Mathematics.*

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Teachers

Many teachers in Alberta, by experimentation and study, have contributed to the understanding of the new programs in mathematics and grateful thanks is extended for the generous assistance they provided. Special acknowledgment is made to the following teachers of Grade VII experimental classes.

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OBJECTIVES

The objectives for mathematics are:

1. To develop an understanding of mathematical concepts.
2. To develop skill in the use of the fundamental processes.
3. To develop systematic methods of analyzing problems and of presenting their solutions.
4. To develop habits of precise thought and expression.
5. To develop an understanding of the significance and application of mathematics in the modern world.

AUTHORIZATIONS FOR GRADE VII MATHEMATICS

Effective September, 1965, either of the textbooks listed below may be selected for use in Grade VII classrooms.

1. *Exploring Modern Mathematics, Book* 1, Keedy, Jameson, Johnson, Purvis and Atkinson (Holt, Rinehart and Winston of Canada, Ltd.).

2. *Seeing Through Mathematics, Book* 1, Special Edition, Van Engen, Hartung, Trimble, Berger, Cleveland and Evenson (W. J. Gage and Co.).

SUPPLEMENTARY REFERENCES

1. *Teacher's Edition*, *Exploring Modern Mathematics, Book* 1, Keedy, Jameson, Johnson, Purvis and Atkinson (Holt, Rinehart and Winston of Canada, Ltd.).

2. *Programmed Supplement*, *Exploring Modern Mathematics, Book* 1 (Holt, Rinehart and Winston of Canada, Ltd.).

3. *Tests for Exploring Modern Mathematics*, *Book* 1 (Holt, Rinehart and Winston of Canada, Ltd.).

4. *Charting the Course for Arithmetic,* Hartung, Van Engen, Knowles and Gibb (Scott, Foresman and Company).

5. *Response Verification Book, Seeing Through Mathematics, Book* 1, Special Edition, Van Engen, Hartung, Trimble, Berger, Cleveland and Evenson (W. J. Gage and Co.).

6. *Teaching Guide, Seeing Through Mathematics, Book* 1, Special Edition, Van Engen, Hartung, Trimble, Berger, Cleveland and Evenson (W. J. Gage and Co.).

INTRODUCTION

A. ELEMENTARY BACKGROUND

Grade VII teachers should be aware that, for the first time in many schools, the students they will be receiving from the elementary grades will come to them with a different background of mathematics. The *Seeing Through Arithmetic* series of texts, published by Gage and Company, and the *Arithmetic We Need* series of texts, published by Ginn and Company, are authorized for elementary grades in Alberta. An important implication is the responsibility of the teacher to determine the specific background in mathematics of the students in each class. The ideal situation would be that the junior high school mathematics teacher become familiar with the content, language, and approach of the elementary mathematics programs. A suggested reference is *Charting the Course for Arithmetic*, published by Gage and Company for teachers using the *Seeing Through Arithmetic* series of texts in elementary classes. Knowledge of the Grade V and VI programs is important to provide the teacher with a feeling of competence in teaching students already familiar with a different approach and terminology.

Certain specific areas of particular importance to teachers of students with a *Seeing Through Arithmetic* background are discussed here.

1. Spiral Approach

The elementary programs now in use place a much greater emphasis on understanding concepts than has been the case in traditional programs. For this reason, many topics are presented on a spiral basis.

quote

a. The pupil's initial contact with a topic is designed to develop the general idea, and some feeling of familiarity with the relations involved.

b. In the second stage, through the use of many illustrations, the complete topic is developed with a view to understanding, but not with the expectation of complete mastery or proficiency in applying the skills involved.

quote

c. The next return to the topic introduces illustrations for review and places emphasis on student skill and proficiency.

It has always been the case that continuous maintenance, drill and review is necessary. Now it appears essential that teachers of Grade VII also be aware of the stage of development that each topic has attained. Since this may be different from the coverage in previous Grade I - VI programs, a study of the present elementary program is most advisable.

2. Fractions

The student coming to Grade VII must not be expected to have complete competence in computing with fractions. The basic detail has been taught, but the Grade VII teacher must maintain the skills and develop competence on the part of the student. Further teaching of this topic is required in the second half of both of the newly authorized texts.

3. Division

The elementary programs, *Seeing Through Arithmetic,* presents long division by development of the idea of repeated subtraction (*Seeing Through Arithmetic, Book* 6, pp. 20-23), in order to produce better understanding than was achieved using the traditional approach. The format for showing the work involved is somewhat different from the format suggested in the past. Two lessons, each entitled a "Sidetrip," found in *Seeing Through Arithmetic, Book* 6, *pp.* 162-164 and pp. 234-235, describe a transition to the traditional way of doing division. These lessons may have been omitted by some classes, and many students may still be using the repeated subtraction way of writing the computation when they reach Grade VII. It will be necessary for the junior high school teacher to familiarize himself with this algorism in order to help pupils with their difficulties. Three examples are shown below for the teacher's consideration.

Seeing Through Arithmetic Grade VI		**Conventional Long Division**	**Short Division**
		595	595 r 55
72) 42895	500	72) 42895	72) 42895
36000		360	
6895	90	689	
6480		648	
415	5	415	
360		360	
55	595	55	

4. Per Cent

The concept of an ordered pair of numbers has been introduced to most students in the elementary grades to clarify the meaning of per cent. Since this approach was also used extensively for rate problems, comparison problems and fractions, the teacher will find it useful to examine this approach in some detail. Adequate information can be found in *Seeing Through Arithmetic, Book* 5, on the pages listed under the index heading "Ratio"; *Seeing Through Arithmetic, Book* 6, on the pages listed under the index heading "Ratio". *Charting the Course for Arithmetic,* and Unit 4, *Seeing Through Mathematics, Book I, Special Edition.*

The customary three "types" of per cent problems can be presented by using rate pairs. A complete solution is shown for the first example only.

1. John saves 25% of the money he earns. How much will he save of $20.00 he earns in one week?

n, the number of dollars John saves

$$\frac{25}{100} \backsim \frac{n}{20}$$

$$25(20) = 100\,n$$

$$5 = n$$

John saves $5.00.

2. Jack has 20 rabbits. He sold 5 of them. What per cent of his rabbits did he sell?

$$\frac{5}{20} \backsim \frac{n}{100}$$

3. Miss Andrews said that 60% of her pupils were present one stormy day. If 15 pupils were present, how many pupils are in Miss Andrew's class?

$$\frac{60}{100} \backsim \frac{15}{n}$$

B. NEED FOR CONTINUOUS DRILL

In any mathematics program, there is a need for continuous drill and review.

Both authorizations were made to permit time to include skill maintenance as a regular part of the daily schedule, rather than allocating it to a special period or class.

The advice is necessary for two reasons:

1. Teachers, when presenting new subject matter for the first time, may become so concerned about doing a thorough job of presentation that insufficient time may be devoted to routine practice and drill.
2. New programs tend to place emphasis on other areas of mathematics in addition to computational skill.

C. CHANGE OF EMPHASIS IN THE NEW PROGRAMS

As suggested in the previous topic, the Grade VII teacher should be aware that the new authorizations present mathematics instruction as something more than the mere developing of computational skill. It is expected that students will be able to gain some understanding of the structure of mathematics. The recurring characteristics of the various number systems and the repeated applications of certain properties should make mathematics more than the memorization of countless disconnected rules. Because greater emphasis is placed on understanding concepts, it is anticipated that pupil attitude to mathematics will show considerable improvement. Teachers of mathematics must be aware of the wider objectives in the new mathematics programs.

D. IMPORTANCE OF TEACHER PREPARATION

The change to a new text book always requires a greater amount of teacher preparation. This normal requirement is accented in the case of the present transition, because authorizations for Grade VII present a different approach. For this reason it is not considered advisable for the teacher to use consistently a routine lecture-type of lesson, followed by a class work period. Also, the type of lesson which has the class reading the questions in order from the text tends to reduce pupil interest and enthusiasm. A pupil-teacher interaction type of lesson, with the teacher presenting problem situations which lead the students to develop many of the concepts through their own efforts appears most suitable. To present this type of lesson adequately requires careful teacher preparation. The number of illustrations needed to develop a concept will vary with the ability of the class. Similarly, the number of exercises needed to fix the concept firmly in the pupil's mind must be given careful consideration. In short, the teacher in the first year will find that successful teaching requires a heavy expenditure of time and effort in preparation.

E. VISUAL AIDS

Teachers have found that visual aids serve to help students understand certain concepts more clearly. Some of the following materials may be useful in the sections presenting geometric concepts and properties of number systems.

a. a flannel board

b. a pegboard, about 4 ft. x 5 ft., which can be painted with blackboard paint and used to show lines and graphs, or fitted with pegs, golf tees, string or elastic to show plane figures.

c. charts

d. cutouts

e. popsicle sticks or beads for students to use when studying the properties of number systems.

EXPLORING MODERN MATHEMATICS
BOOK I

A. GENERAL CONSIDERATIONS

1. Teacher's Edition

Exploring Modern Mathematics, Book I, Teacher's Edition, is available from Holt, Rhinehart and Winston of Canada, Ltd., as a separate publication. Each teacher using this text is strongly advised to have a copy of this publication. Not only does the "*Teacher's Edition*" contain answers to each of the exploratory exercises and practice exercises, it also presents an explanation of the approach and philosophy adopted in the program. There are valuable suggestions made on how to use the exploratory exercises and how to maintain skills and understanding. A useful overview of each chapter is provided as well as exercise-by-exercise, lesson-by-lesson hints and suggestions. As a guide to the program, the "*Teacher's Edition*" is a valuable asset to the teacher.

2. Semi-Programmed Nature

The development of new concepts in this program is provided for by individual student experience. A series of questions, or the order of the problems presented in each section, helps the student to gain his own discovery of the mathematical concept required.

Use of the "Let's Explore" Exercises

Exercises entitled "Let's Explore" are for the specific purpose of allowing the student to discover mathematical ideas on his own. THESE EXERCISES ARE NOT TO BE GRADED. It is considered essential that the teacher using the text, "*Exploring Modern Mathematics, Book I*," for the first time, work through the "Let's Explore" exercise before class in order to anticipate problems which the students may encounter.

It is also the teacher's responsibility to determine whether certain topics in the exploratory exercises may be deleted for superior students. In some instances, a short discussion may replace such topics. Teachers who have taught this program have found that in general the exploratory exercises make a useful, stimulating and vital contribution to their teaching. Certain exercises may be difficult for the average student, and it will be necessary for the teacher to ensure that the student has understood the topic presented.

Experience has shown that it is useful to spend a short time at the beginning of the class period to make the student aware of a gap in his knowledge and to create some curiosity about the exploratory exercise to follow. The class should then proceed, on their own, to work through the exercise, the teacher being careful to give assistance only to those students who are obviously making no progress. Students who finish first might be encouraged to do appropriate questions in the practice exercise which usually follows. It is important to bring the class together following the exploratory exercise to discuss the concepts under consideration. On some occasions, it will be necessary for the teacher to work at the blackboard with the entire class, developing the concepts again and reviewing the conclusions.

The assignment of a "Let's Explore" exercise as homework has been found satisfactory by teachers, since experience indicates that individual differences within classes are too great, and the teacher must be available to provide assistance. However, teachers who have taught this program found that most students can do the exploratory exercise in class, and that sufficient understanding can be gained for students to begin the practice exercises, which can then be finished by homework.

3. Programmed Supplement

The student who has been absent for a lengthy time or who is having difficulty understanding the concepts may find the textbook unsatisfactory because this text was not designed to allow a student to work completely on his own. A *"Programmed Supplement", Exploring Modern Mathematics, Book I,* is available which contains the "Let's Explore" exercises with answers developed in more detail, using a workbook-type format. Teachers may find that having several copies of this publication for students' use will help to avoid undue repetition of lesson content previously taught.

4. Schedule of Units and Minimum Program

a. The following approximate time schedule appears to be suitable for most students.

Chapter 1	3 Weeks
Chapter 2	4 Weeks
Chapter 3	3 Weeks
Chapter 4	5 Weeks
Chapter 5	3 Weeks
Chapter 6	6 Weeks
Chapter 7	4 Weeks
Chapter 8	4 Weeks
Chapter 9	3 Weeks
	35 Weeks

b. Content that may be omitted for classes or individual students of moderate ability.

The sections listed below include those which are marked "Optional" in the textbook and those which teachers have found may be omitted for slow learners. Exercises identified with an asterisk (*) in the textbook are related to the "Optional" sections.

Chapter 1. Multiplication Using Base Five Numerals p. 26
Other Numeration Bases p. 28
Section 1.9 Let's Explore pp. 28-29
Exercises pp. 29, 30, 31
Review Exercises p. 33, 13f, 13g; p. 34, 16

Chapter 4. Factoring and Prime Numbers

Section 4.18	Let's Explore pp. 160-161.	Proof that the sum of two even numbers is even.
	Exercises p. 163, 3, 4, 5	
4.19	Let's Explore pp. 163-164.	Proof that the set of odd numbers is not closed under addition.
4.21	Let's Explore pp. 165-166. The Euclidean Algorithm p. 166.	Proof that the set of odd numbers is closed under multiplications.
4.22	Let's Explore pp. 166-167.	
4.23	Let's Explore pp. 167-168. Exercises p. 168	

4.24 Let's Explore pp. 168-169.
4.25 Let's Explore pp. 169-170.
4.26 Let's Explore pp. 170-171
4.29 Let's Explore pp. 173-171. Divisibility 9.
Exercises pp. 174, 1, 2.
4.30 Let's Explore pp. 174-175. Divisibility by 3.
Exercises pp. 175-176, 1, 2, 3, 4.
Exercises pp. 176-177, 1, 2, 3, 4.

Chapter 5. How to Make and Study Mathematical Systems
Section 5.5, 5.6, 5.7, 5.8, 5.9, pp. 196-204.
5.22 Let's Explore pp. 218-219. Proof for additive inverses
Exercises pp. 219, 2, 5.
Modular Arithmetic pp. 219-220.
Exercises p. 220, 1, 2, 3, 4.
Mathematical Systems Without Numbers p. 220
5.23 Let's Explore pp. 220, 221, 222.
5.24 Let's Explore pp. 222.
Exercises pp. 222, 1, 2.

5. Sequence of the Program

Only after considerable experience with the text, and after even more careful thought, should the teacher contemplate changing the sequence of the course. It is further suggested that teachers do not omit any section other than those marked "Optional" and those suggested in this curriculum guide.

Teachers presenting this course for the first time may find it useful to omit all sections marked "Optional" for "average" and "below average" classes, in order to complete the program.

6. Proofs and Definitions

Teachers who have taught this program have found that the development of proofs offered in the textbook was not suitable for many students. Strong interest in this type of mathematical rigor should not be expected from students at this grade level.

Most of the concepts and definitions that appear at the end of the exploratory exercises need not be committed to memory. The ability to verbalize concepts should be encouraged. However, the teacher must place emphasis on understanding and using a concept, rather than memorizing a definition.

7. Review Exercises

The review exercises at the end of each chapter are not intended to be used only when the end of the chapter has been reached. It is suggested that better use of these exercises can be made by assignments given to the student when a particular section or topic has been taught. Most chapters can be divided into several sections for this purpose.

Teachers are referred to the "*Teacher's Edition, Exploring Modern Mathematics, Book I,*" page XV, for further suggestions on the use of review material.

B. LESSON COMMENTS

Chapter 1 — Numeration Systems

This chapter develops a review of the basic multiplication and addition facts, presents the concepts of decimal place values, and develops an appreciation of the historical aspects of mathematics and its place in society.

The commons below refer to the topics presented on the pages listed in this chapter.

Page 4 — Egyptian Numeration

Page 5 — Babylonian Numeration

There is no need for students to remember the symbols used in these two numeration systems.

Page 18, 19, 20 — Changing a Decimal Numeral into a Base Five Numeral

Two methods are suggested for this procedure. The method explained on page 18, in which the student is encouraged to think in terms of groups of 1, 5, 25, 125, etc., appears to have more meaning for students at this level. At least two class periods may have to be used for this topic.

Page 26 — Numerals for Numbers Less Than 1

Students require considerable practice with this topic, in order to understand expanded numeral notation and place value in decimal numerals. Pages 261 and 262 of this textbook teach this topic, and the teachers may find it useful to use these exercises at this time. An extra class period may have to be used for this purpose.

Page 28 — Other Numeration Bases

Slower learning classes might be well-advised to omit this section.

Chapter 2 — Properties of Whole Numbers

This chapter begins the study of the properties of whole numbers.

Pages 39, 40 — The Symbol %

Per cent is described in two ways in this textbook. The first description is a comparison, and the second is that of another name for a number. Students with a background of the "*Seeing Through Arithmetic*" program in the elementary grades will be familiar with the ratio notation used on page 39. Teachers are referred to pages 4 and 5 of this curriculum guide for further information about this approach. Both approaches used in this textbook appear to have some merit.

Page 76 — Before starting the section "Multiplying By Ten," the class should do questions 1 to 11 in the Review Exercise on pages 91 and 92. Students will likely be in need of some consolidation and review at this stage. Students should be familiar with the properties presented thus far, and be able to name them.

Page 76 — Multiplying By Ten

Most classes will benefit by considerable oral drill or "mental arithmetic" with this method.

Chapter 3 — Points, Lines, Curves and Planes

The geometry presented here uses the "point-set" approach, is non-metric, and is basically Euclidean in content. The set concept helps to unify ideas in geometry, and helps to relate geometry to the rest of mathematics.

The exploratory exercises lead students to develop notions of point, geometric figure, segment, line, half line, intersections of sets, lines on a point, planes, planes on a line, separations, curves and polygons.

Students are encouraged to use precise language and expression.

Chapter 4 — Factoring and Prime Numbers

This chapter leads the student into informal proof, and many of the concepts presented are useful in later parts of this program.

Teachers are referred to page 8 of this curriculum guide for specific suggestions for topics which might be omitted for slower learning students.

Chapter 5 — Mathematical Systems

This chapter emphasizes the structure of mathematical systems. These patterns are to be found in algebraic topics in later parts of the program, and are very important to the student's preparation for further mathematics.

It is suggested that a review of the commutative, associative, closure, distributive and identity properties of the whole number system be made as an introduction to this chapter.

Page 197 — How to Make and Study Mathematical Systems

Teachers who have taught this program suggest that most students may be well-advised to omit the material included on pages 196 through 204. Superior students will be well-challenged by this material. Section 5.10, Let's Explore, p. 204-205, is difficult, and the teacher might simply discuss the idea of inverse elements with the class, using the topic note at the bottom of p. 204, for the topic, "Inverses in Clock Arithmetic", p. 206.

The topic, "Inverses in Clock Arithmetic", is the introduction to negative numbers.

Page 211 — Multiplication in Clock Arithmetic

Exercises, p. 211, questions 5 and 6, will likely require some explanation by the teacher since the idea of multiplicative inverses and identities will not be familiar to the student as yet.

Pages 219, 220 — Modular Arithmetics

Average and above average students will benefit from a study of modular 6 arithmetic since it provides excellent reinforcement to the previous topics. Slower learning students might be well advised to omit this section.

Page 220 — Mathematical Systems Without Numbers

This can be treated as an optional section.

Chapter 6 — The Number System of Arithmetic

This chapter includes much incidental drill on the fundamentals of arithmetic while presenting arithmetic numbers as a system with a set of elements and one or more operations defined on the set. Properties are discovered and proved with respect to these operations, and rules for calculating are then developed. Page 261 — Before students begin this exercise, it is suggested that questions 5, 6 and 7 of the Review Exercises, page 283, be assigned.

Pages 267 and 268 — Multiplication Using Decimal Numerals

Two methods for locating the decimal point in a product are explained in this topic, the first by dividing by the appropriate power of ten and the second by estimating. This program recommends that emphasis should be placed on estimating the place of the decimal point, but some teachers have found that many students in Grade VII cannot estimate with sufficient accuracy to justify this emphasis. Both methods should be employed, but it is suggested that the teacher be reasonably certain that students understand the method employing division by the required power of ten, in order that an accurate method be available for use in computation.

Chapter 7 — Figures on a Plane

Through the measurement of one-dimensional figures, the classification of the basic sets of points is taught. Standard measures are introduced, and the distinction between a set of points and a number is made clearly.

Chapter 8 — Measures

Systems of measures are studied in this chapter, with emphasis on how measures are developed and problems associated with measurement.

Many of the topics presented in this chapter will be simplified if a "ratio method" of problem solving is adopted at the outset. While this may mean that some of the exploratory exercises will not be useful, the approach will be justified in view of the unity and clarification it brings to the problems. The use of the rate pair has such a wide application that the initial effort required will be worthwhile. Typical problems solved by this approach appear below, and the teacher is referred to pages 4 and 5 of this guide for further information.

Page 348 — British Units of Length

Question 2a. How many inches are there in 4½ feet?

x, the number of inches in 4½ feet.

$$\frac{x}{4\frac{1}{2}} = \frac{12}{1}$$

Note that if this statement is to be true, then the cross-products must be equal.

$$x \cdot 1 = 4\tfrac{1}{2} \cdot 12$$

$$x = 54$$

Answer: There are 54 inches in 4½ feet.

Pages 348, 349 — The Metric System

Once the metric system has been explained, most students will need considerable drill on the relationships involved.

Page 350

Question 5a. 7.2 cm. = ―――――― mm.

x, the number of millimeters in 7.2 centimeters

$$\frac{7.2}{x} = \frac{1}{10} \qquad (1 \text{ cm.} = 10 \text{ mm.})$$

$$7.2 \cdot 10 = x \cdot 1$$

$$72 = x$$

Answer: 72 mm.

Page 362 — Capacity

Page 363

Question 3a. 9 gal = ―――――― pt.

x, the number of pints in 9 gallons.

$$\frac{9}{x} = \frac{1}{8} \qquad (1 \text{ gal.} = 8 \text{ pints})$$

$$9 \cdot 8 = x \cdot 1$$

$$72 = x$$

Answer: 72 pints

Page 367 — Mass

Question 1. Assume that gravity on the moon is 1/6 of that on the earth.

A man weighs 160 pounds on earth. What would he weigh on the moon?

n, the number of pounds a man weighs on the moon.

$$\frac{1/6}{1} = \frac{n}{160}$$

$$1/6 \cdot 160 = 1 \cdot n$$

$$26\ 2/3 = n$$

Answer: 26 2/3 pounds.

Page 371 — Measures of Time

Question 6c. 3 years = ______ days

1 year = 365 days

n, the number of days in 3 years.

$$\frac{3}{n} = \frac{1}{365}$$

$$3 \cdot 365 = n \cdot 1$$

$$1035 = n$$

Answer: 1035 days

Pages 379 - 383 — Changing Units

The use of ratios is an alternate and very useful method of solving problems in this section.

For example,

Page 380, Section 8.14

Question 2. Change 3 feet to inches.

x, the number of inches in 3 feet.

$$\frac{3}{x} = \frac{1}{12}$$

$$3 \cdot 12 = x \cdot 1$$

$$36 = x$$

Answer: 36 inches.

Page 382, Example 3.

Change $\frac{3 \text{ ft.}}{\text{sec.}}$ to $\frac{\text{yd.}}{\text{min.}}$

It is suggested that this problem be solved in two stages, since two units must be changed.

(a) $\frac{3 \text{ ft.}}{\text{sec.}} = \frac{\underline{\qquad\quad} \text{ yds.}}{\text{sec.}}$ Convert ft. to yds. $\frac{3}{1} = \frac{3}{n}$ $n = 1$

$\therefore \frac{3 \text{ ft.}}{\text{sec.}} = \frac{1 \text{ yd.}}{\text{sec.}}$

(b) $\frac{1 \text{ yd.}}{\text{sec.}} = \frac{\underline{\qquad\quad} \text{ yd.}}{\text{min.}}$ Convert secs. to mins. $\frac{60}{1} = \frac{1}{n}$ $n = \frac{1}{60}$

$\therefore \frac{1 \text{ yd.}}{\text{sec.}} = \frac{1 \text{ yd.}}{1/60 \text{ min}}$ or $\frac{60 \text{ yds.}}{\text{min.}}$

Page 383

Question 13. $\frac{30 \text{ mi.}}{\text{hr.}} = \frac{\underline{\qquad\qquad} \text{ ft.}}{\text{sec.}}$

The problem may be solved in three stages as follows:

(a) $\frac{30 \text{ mi.}}{\text{hr.}} = \frac{\underline{\qquad\quad} \text{ ft.}}{\text{hr.}}$ Convert mi. to ft. $\frac{1}{5280} = \frac{30}{n}$, $n = 30 \times 5280$ ft.

$\therefore \frac{30 \text{ mi.}}{\text{hr.}} = \frac{\underline{\qquad\quad}\ 30 \times 5280 \text{ ft.}}{\text{hr.}}$

(b) $\frac{30 \times 5280 \text{ ft.}}{\text{hr.}} = \frac{\underline{\qquad\quad} \text{ ft.}}{\text{min.}}$ Convert hrs. to mins. $\frac{1}{60} = \frac{1}{n}$, $n = 60$

$\therefore \frac{30 \times 5280 \text{ ft.}}{\text{hr.}} = \frac{30 \times 5280 \text{ ft.}}{60 \text{ sec.}}$

(c) $\frac{30 \times 5280 \text{ ft.}}{60 \text{ sec.}} = \frac{\underline{\quad} \text{ ft.}}{\text{sec.}}$ Convert mins. to secs. $\frac{1}{60} = \frac{60}{n}$ $n = 3{,}600$

$$\frac{30 \times 5280 \text{ ft.}}{60 \text{ min.}} = \frac{30 \times 5280 \text{ ft.}}{3600 \text{ sec.}} = \frac{44 \text{ ft.}}{\text{sec.}}$$

Three stages were illustrated here to show a solution which is based on the relationships between the units of measure involved in the problem. Whether the method used in the textbook or a method using a "ratio" approach is used, it must be emphasized that the determination of a solution must be understandable to the student and not a calculating device used only to reach a correct answer.

Page 384 — Changing from Metric Units to British Units

Question 6b. 41.2 m. = _______ in.

x, the number of in. in 41.2 m.

$$\frac{41.2}{x} = \frac{1}{39.37}$$

$$x \cdot 1 = 41.2 \cdot 39.37$$

$$x = 1622$$

Answer: 1622 inches.

Page 385 — Per Cent Questions

Per cent questions of the three usual types are easily handled by using ratios without differentiating between types. Refer to pages 4 and 5 of this curriculum guide for examples of problems.

Chapter 9 — Applying the Number System of Arithmetic

Pages 391 - 398 — Interest

Although the ratio method leads to a good understanding of the formula $i = prt$, teachers may prefer to follow the procedure as presented in the textbook in Chapter 9.

Pages 400 - 404

It is suggested that the ratio method be considered an alternate method in solving problems concerning discounts, rates of discounts and commissions. Again it must be emphasized that the understanding of the relationships involved in problem situations is important, in order to avoid having students reach conclusions not justified by the procedure used.

SEEING THROUGH MATHEMATICS, BOOK I, SPECIAL EDITION

A. GENERAL CONSIDERATIONS

1. Teaching Guide

Teaching Guide, Seeing Through Mathematics, Book I, Special Edition, is available from Gage and Company as a separate publication. In day-to-day preparation, constant reference to the *Teaching Guide* is very desirable because the overall purpose of each lesson is outlined, background concepts are presented, and teaching suggestions are given. It is useful to spend some time studying "Introductory Remarks," pp. 10-40, before meeting classes for the first lessons. The "Lesson Notes", pp. 42-299 give suggestions for the individual lesson and, during teacher preparation, might well be read in conjunction with the text.

2. Semi-Programmed Nature

The *Seeing Through Mathematics* textbook, together with the *Response Verification Book*, describes detailed lessons useful to the teacher. However, no single form of presentation is best for all teachers, all classes, or for all topics. Therefore, the teacher must use initiative in deciding upon a particular approach. To expect the student to read, or listen to the teacher read, every question in the text would soon destroy pupil interest. The detail as shown in the text should be considered as a suggested approach, and many lessons will secure best pupil development if the teacher is prepared to the point that the textbook can be put aside and the lesson presented in the manner the textbook suggests. In general, experience with the experimental classes would seem to indicate that pupils cannot be expected to handle the developmental content of the lessons completely as seat assignments or as homework assignments.

The Response Verification Book, Seeing Through Mathematics, Book I, Special Edition, is a separate publication giving expected replies to all the questions in the lesson development and to the exercises which normally occur at the end of each lesson. Explanatory comment is also inserted where necessary. Teachers should have a copy available for their own use, and many schools may desire to have copies for student use, but not for the student's constant reference. In the preparation of lessons teachers will find the *Response Verification Book* most helpful. Pupils who have been absent or pupils who have entered Alberta schools from other provinces may use it to learn material the class has been taught in their absence.

3. Minimum Program and Schedule of Units

Each of the seven units which constitute the authorized program, and the 97 lessons contained therein, do not require equal blocks of class time. Completion of the course necessitates a rate of progress approximating two lessons completed each three days. Ninety-seven lessons to be completed in about 200 school days leaves some margin for testing, for extra drill, and for longer lessons.

To avoid using too much time for any one unit, teachers who have had experimental classes suggest the following timetable, which includes all testing and review. It should be noted that the maximum time cannot be used by a class for every unit, since the number of weeks in the school year is less than 42.

	Minimum		Maximum	
Unit 1	5	-	7	Weeks
Unit 2	3	-	4	Weeks
Unit 3	4	-	5	Weeks
Unit 4	3	-	5	Weeks
Unit 5	2	-	4	Weeks
Unit 6	4	-	6	Weeks
Unit 7	9	-	11	Weeks
TOTAL	30	-	42	Weeks

Only faster moving classes are likely to complete the total content of the text and maintain the schedule suggested above. Unit 7 is considered essential for the work in future grades, and class time must be budgeted to allow for the completion of this unit.

Deletions necessary should be selected from the following list, according to the abilities of the class, so that all students may complete the work of the last unit.

CONTENT THAT MAY BE OMITTED BY CLASSES OR INDIVIDUAL STUDENTS OF MODERATE ABILITY

NOTE: Exercises headed "Special Challenge", designed to challenge the most competent student, should be used for enrichment only for students well advanced in relation to the above schedule.

Unit	Lesson	Content
Unit 2	28	Congruent triangles: displays 5 and 6 and exercises A through G on page 111.
Unit 3	39	Compound conditions for more complex problems. Conditions for abstract problems.
Unit 4	48	Abstract problems involving rate pairs.
Unit 5	49	Tally, code, and grouping numeration systems.
	50	Numeration systems that have place value.
	51	Place value and digits in numeration systems.
	52	Egyptian, Roman and Babylonian numeration systems.
	54	Converting numeral from one base to another.
	55	Computing in numeration systems other than the decimal system.

Unit	Lesson	Content
Unit 6	62	Development of the commutative properties through set considerations; displays 2 and 3 and exercise B through K on page 265; displays 6 and 7 and exercises N through V on page 266.
	64	Proof of the right distributive property; displays 4 and 5 and exercises O through U on pages 275-276.
	67	A finite number system
	68	Identity elements developed by means of set ideas: displays 1, 4 and 5, exercises A through G on page 289, and A through H on page 290.
	69	Development of the notion of twin primes, thinking of even numbers as the sum of two primes, and the number of primes between a given number and twice the given number: displays 5 and 6 and exercises A through I on pages 297 and 298.
Unit 7	72	A proof for the reduction property of fractions: display 3 and exercises A through M on pages 313-314.
	80	A proof for the commutative property of multiplication in the system of rational numbers of arithmetic: display 4 and exercises I through N on page 349.
	82	A finite number system: display 12 and exercises G through J on page 359.
	83	The identity elements.

4. Sequential Nature of the Program

In this program, a sequential development of concepts from lesson to lesson and from grade to grade has been prepared. For this reason it is inadvisable for the teacher to attempt modifications in order, or approach, at least until he has become familiar with the total program. If the teacher neglects to adopt the new methods and symbolism as presented in the text a serious hardship may be imposed on the pupil in later lessons.

5. Use of Definitions

Students are expected to have an intuitive grasp of the definitions used in the textbook. Pupils must not be expected to memorize these definitions per se. A "working understanding" is the most that should be expected of the Grade VII student.

B. LESSON COMMENTS

Lessons marked with an asterisk (*) may be omitted, or may have some content which may be omitted, by slow learners. Refer to pages 18 and 19 of this curriculum guide.

UNIT I — Sets, Conditions and Variables

Lessons 1 - 7

The teacher should study this block of lessons before presenting Lesson 1. The groundwork for the concepts in this section began in the elementary grades, and these lessons formalize the concepts and develop a useful symbolism. Most students need the long introduction to this unit. Definitions should not be memorized, but emphasis should be placed on understanding the meaning of the definitions and using them to help understand problems.

The notation for the standard description of sets is a difficult one for students, since at this stage the standard description does not shorten the presentation of ideas. However, this notation is very useful in later lessons, and must be taught well.

Precision of language should begin immediately with the teacher using mathematical terms correctly, and encouraging the pupil to do the same. Distinctions are made in these lessons between a condition for equality and an equation and the terms are not used with the traditional connotation in all cases. Students soon learn to use mathematical terms correctly if the teacher insists upon and practices precise expression.

The apparently tedious writing of sentences which occurs when a placeholder is being replaced by members of a universe serves to develop a clear understanding of the concepts and should not be terminated until the teacher feels confident that the class has grasped the ideas clearly.

Lessons 8 - 12

An approach to geometry completely different from the traditional approach has been introduced in these lessons. Because geometric figures are defined as sets of points, this approach is called the point-set approach to geometry. The ideas of sets, and the set notation introduced in lesson 1 - 7 are now extended to include geometric concepts.

Standard description of sets are used again, and the student will need practice in using this notation with the geometric content.

Also, precision of terminology and consistency in work usage will require constant effort on the part of the teacher.

Lesson 13 — Number, Lines and Graphs

One-to-one correspondence, which was used in many parts of the elementary program, is used in this lesson to associate a number set with a point set. This lays a careful foundation for graphical representation in later lessons.

Lessons 14 - 16

These lessons present a review of the operations of multiplication and division as presented in the *Seeing Through Arithmetic* elementary program. The successive subtraction approach to division was introduced to improve understanding. As the children constantly refine their ability to estimate quotients the format becomes closely analagous to the traditional way of writing the operation of division. Some teachers recommend that a transfer to the more common algorism be made at this time. A suitable approach is shown in *Seeing Through Arithmetic,* Grade VI, pp. 162 - 164 and pp. 234 - 235. A change at this time may have advantages in later lessons where decimals are used.

Lessons 17 - 21

These lessons use conditions for equality and inequality and extend the problem-solving technique begun in the elementary grades. Problems in these lessons involve additive and subtractive situations only, and care should be taken not to confuse the student by introducing other situations. Conditions for inequality can be solved by reference to the appropriate conditions for equality.

UNIT 2 — Intersection and Union of Sets

Lessons 22 - 23

The operations with sets outlined in these lessons are fundamental to later work in algebra and geometry. Venn diagrams are useful in clarifying the operations of intersection and union of sets.

Lessons 24 - 25

These lessons apply the concepts of Lessons 22 and 23 to the solution of problems.

Lessons 26 - 29

The operations with sets are applied to the consideration of geometric figures in these lessons. It may be necessary to review the content of Lessons 8 to 12 before introducing this block of lessons. The geometry is completely non-metric. This necessitates definitions completely different from traditional definitions. The idea of subsets of a plane constituting regions will be new to many teachers and will need careful study.

UNIT 3 — Conditions in Two Variables

The concept of ordered pairs of objects is fundamental to many areas of this program. In this unit, the components of the ordered pairs are used to find solution sets for conditions that have two variables.

Lesson 30 — Ordered Pairs

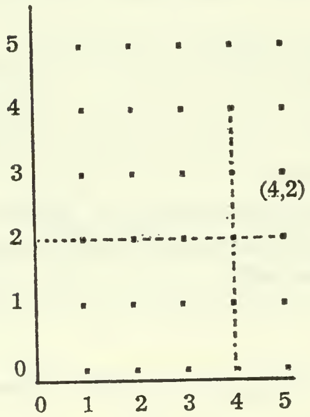
The concepts of this lesson are so basic for the development of the *Seeing Through Mathematics* program that a complete understanding should be gained and a MASTERY of the concepts related to ordered pairs should be evident in the exercises done by the student.

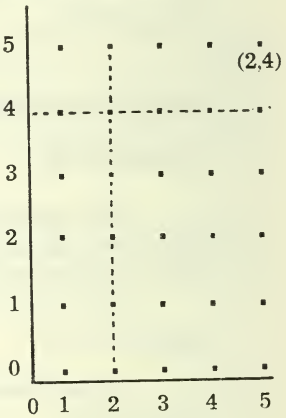
Lesson 31 — Sets of Ordered Pairs

A thorough explanation of the generation of Cartesian sets must be given, so that students will understand clearly the matching of one set in turn with each member of a second set. The order of the components of an ordered pair must be emphasized by the teacher.

Lesson 32 — Graphs of Sets of Ordered Pairs

Graphs of sets of ordered pairs are used in this lesson to show the solution sets of conditions in two variables. Reference may be made to Lesson 13, where a one-to-one correspondence between a number and a point on a line is used. Again, the student must realize that the order of the components of a given pair is very important in determining the position of the point in the plane that is associated with that pair. Some students, because of perceptual differences, may not see this relationship clearly unless the teacher presents a clear set of diagrams for this purpose.





Lessons 33 and 34 — Conditions, Graphs, and Cartesian Set N X N

Using a graph to find a solution set, finding a condition from the graph of a solution set, and using a standard description to name a solution set of a condition in two variables are taught in these lessons. The development in the text is detailed and will provide an excellent basis for presentation.

Teachers should be alert to the usefulness of relating, in terms of the graphical representation, the solution sets of conditions for inequalities and the solution set of the associated condition for equality.

Lesson 35 — Charts and Solution Sets

Two charts, one for conditions for equality and one for conditions for inequality, are used to find solutions of conditions in two variables. By replacing one of the variables, the student can then determine the replacement for the other variable that makes a true statement from a condition.

$U = N X N$

$x + y = 4$

x	0	1	2	3	4
y	4	3	2	1	0

$U = N X N$

$x + y < 5$

x	0	1	2	3	4
y	4	3	2	1	0
	3	2	1	0	
	2	1	0		
	1	0			
	0				

Extensive drill should be given so that a thorough grasp of the use of the charts is attained. Point out to the student that there is an exact relationship between the reading of the charts and the ordered pairs included in the solution set.

Lesson 36 — Compound Conditions in Two Variables

Achievement in this lesson is dependent on the successful mastery of the ideas involved in the two previous lessons.

Lessons 37, 38, 39 and 40

These lessons present simple and compound conditions in two variables, and are the culmination of the previous lessons.

The student must learn to use computation and make generalizations in order to do these problems quickly and correctly. Generalizations concerning which ordered pairs N X N are members of the solution set of a given condition can frequently be made by observing patterns in other conditions that resemble it. For example, if the solution set of $y = 4x + 3$ has been found, it can be used to advantage in finding the solution set of $y > 4x + 3$ and the solution of $y < 4x + 3$.

The conditions for equality or inequality found in the problems do not require a formal technique for solving simultaneous conditions to find solution sets.

UNIT 4 — Conditions Involving Rate Pairs

Lesson 41 — Equivalent Ordered Pairs

This lesson shows that a rate can be determined from a particular type of physical situation, that ordered pairs are used to represent rates, and that many ordered pairs can be used to represent the same rate.

Attention is drawn to the definition of a proportional relation through the use of equivalent ordered pairs. Two ordered pairs (a, b) and (c, d), are equivalent if, and only if, $a \times d = c \times b$ is a true statement and the universe is the set of counting numbers.

Lesson 42 — Rate Pairs and Ratios

A rate pair is a member of a proportional relation. The symbol that refers to a rate pair is a ratio, just as the symbol for a number is a numeral.

Standard descriptions are useful in this lesson, since proportional relations are infinite sets unless the chosen universe is finite.

Lesson 43 — Solution Sets of Conditions for Equivalence

In this lesson, students are required to find the missing component of a rate pair that is equivalent to another rate pair. The definition of equivalent ordered pairs is used. For example,

$$\begin{aligned} 7/2 &\sim 28/n \\ 7n &= 2(28) \\ n &= 8 \end{aligned}$$

More formal techniques for finding the solution set of a condition of equality are presented in later units. In this lesson, the teacher should emphasize a method based upon the students' elementary background. Thus, $7/2 \sim 28/n$ will be equivalent only if the replacement for n makes $7n = 56$ a true statement.

Lesson 44 — Problems Involving Conditions for Equivalence

Consistency in expressing rate pairs in this lesson is very important. If the first component, 7, in the relation $7/2 \sim 28/n$, refers to the number of things bought, then the first component 28, of the second pair should also refer to the number of things bought. Similarly, if the second component of the first pair, 2, refers to cents, then the variable, n, for the second component of the second pair must also refer to cents. Note also that, when writing a condition of equivalence, the units or names of the number of things are not stated. An example follows:

John purchased 7 nails for 2 cents. At the same rate, what is the cost of 28 nails?

n, the number of cents 28 nails cost.

$7/2 \sim 28/n$

$7n = 56$

$n = 8$

28 nails cost 8 cents.

The solution of the condition of equivalence, $2/7 \sim n/28$, will also result in correct solution to the problem. However, it is customary to state the condition of equivalence in the order of mention of the rate pairs expressed in the problem. In particular, slower students will benefit by a consistent approach to the order of writing the condition.

Lesson 45 - 46 — Per Cent, and Problems Involving Per Cent

Rate pairs may be used to represent comparisons as well as rates. A per cent may be defined as a rate pair that represents a comparison and whose second component is 100. Thus a per cent is a special kind of rate pair.

In the initial development of lesson 45, the social applications of arithmetic are omitted. A thorough explanation of common business terms used in the problems is advisable.

The commonly used "three types of per cent" problems can be taught using an equivalence of rate pairs without distinguishing types.

Lesson 47 — Problems Involving Compound Conditions About Rate Pairs

A review of Lesson 41, pp. 167-72, and of Lesson 46, pp. 190-193, may be necessary before beginning this lesson. Note that if one of the conditions in a problem has only one variable, it should be solved first. An apt student might see the possibility of eliminating one of the variables by substitution. Such exploratory methods should be encouraged if they are mathematically correct.

Question 13, page 198, is a unique problem in this book because it introduces three variables before the concept of an ordered triple has been introduced. The universe for each variable in this problem is C where $C = \{1, 2, 3, \ldots\}$, and not $C \times C \times C$.

Lesson 48 — Abstract Problems Involving Rate Pairs

This lesson requires the solution of problems which include rate pairs in the solution sets.

The Applying Mathematics exercises, pp. 201-2, use some of the concepts of business arithmetic, and it may be necessary to give considerable explanation of the terms used before students can be expected to understand the mathematical situations.

Lesson 49 — Tally, Code and Grouping Numeration Systems

Some explanation at this point of the difference between a number system and a numeration system will prove beneficial later. (See *Teaching Guide, Seeing Through Mathematics, Book I, Special Edition.*)

When discussing these three simple systems of numeration, the properties of repetition, addition, and base should be developed well. They are unifying concepts to which reference is made in later lessons.

It is not necessary that the symbols used to develop the grouping system be memorized or copied exactly.

Lesson 50-51 — Numeration Systems That Have Place Value; Place Value and Digits in Numeration Systems

The teacher might prefer to combine these two lessons into a single presentation. The "On Your Own" exercise, page 214, could be completed orally under this arrangement.

Although it is possible to use different symbols to represent the numbers ten and eleven in a system whose base is greater than eleven, it is wise to use the symbols and names given, since they are used again in later lessons.

Lesson 52 — Egyptian, Roman and Babylonian Numeration Systems

The purposes of this lesson are to present a review of the properties of addition, repetition, base, and place value, and to lead students to an understanding of the evolution of the decimal system used today. With the exception of the Roman system, which has some practical significance, students should not be required to memorize the symbols and details of these ancient systems.

It is suggested that no more than two class periods be spent on this lesson.

Lesson 53 — Factors, Powers and Exponential Notation

The ideas presented in this lesson are basic to much future work. While it is not necessary that the formal definitions be memorized, students should be expected to master the ideas thoroughly.

Lesson 54-55 — Converting Numerals from One Base to Another; Computing in Numeration Systems Other Than the Decimal System

It is very easy to spend too much class time on these two lessons. The purpose in presenting these lessons is to develop a better understanding of the decimal system. Emphasis should be placed on basic notions that can lead to a better understanding of operations in the decimal system ("carrying", for example). It is not important that students be able to manipulate the symbols well.

It is suggested that no more than four class periods be spent on these two lessons.

Lesson 56 — Decimal Numerals for Numbers Greater Than One Thousand

Students should be expected to master the ideas in this lesson, since they are used in future work.

UNIT 6 — The Natural Number System

The general objectives of this unit are to develop the properties of natural numbers and to define a number system. It may see at first glance that the text merely applies unfamiliar names to familiar concepts. These concepts, however, apply to all the number systems that the student will meet in his future study of mathematics.

Lesson 57 — Equivalent Sets

The ideas of "mapping" and "one-to-one correspondence" are the first building stones for the development of the natural number system. It is useful at this stage to have the students imagine that they must start from the ideas of "one-to-one correspondence" and "set" and build a complete number system. The concept of one-to-one correspondence leads to the idea of equivalent sets, which in turn leads to the ideas presented in the following lessons.

Lesson 58 — Natural Numbers as Properties of Sets

In this lesson a natural number is developed as the common property of equivalent sets. It is important that students grasp the idea that a natural number is a common property of a standard set and all sets that are equivalent to the standard set.

Lesson 59 — Ordering of the Natural Numbers

In this lesson a rule is developed whereby the natural numbers can be ordered. Since the system is being built from the beginning such order has not yet been established. The concept of proper sub-set leads directly to the definition of "less than", which in turn gives rise to the order and successor properties.

A Comment Concerning Lessons 60 - 64

The characteristics of a number system are as follows:

1. There is a set of objects.
2. Two operations are defined for this set.
3. Each operation has the closure property.
4. Each operation is commutative.
5. Each operation is associative.
6. One of the operations distributes over the other.

By the end of Lesson 59 a set of objects (numbers) has been developed and ordered. Lessons 60 to 64 develop understanding of the properties associated with the two defined operations, addition and multiplication. It is suggested that these five lessons be studied in a block by the teacher before individual lessons are prepared.

The reason for the study of these properties will become evident as pupils meet other number systems.

Lesson 60 — The Closure Property of Addition

The idea of addition as a special many-to-one mapping of the set of ordered pairs of natural numbers on to the set of natural numbers is sometimes difficult for students to grasp. This idea should be developed through the use of numerous examples. The *Teaching Guide, Seeing Through Mathematics, Book I, Special Edition,* pp. 160 - 162, presents a good discussion of this section.

The concept of closure is a simple but important idea.

The relation of addition to subtraction as presented in the latter part of the lesson should be developed fully, since it has wide use later.

Lesson 61 — The Closure Property of Multiplication

In this lesson the second operation, multiplication, is defined through the use of the Cartesian set of two given sets. Binary operation, many-to-one correspondence, and closure are reviewed.

If the students have a good grasp of the previous lesson this lesson can be covered quickly.

Lesson 62 — The Commutative Properties

It is suggested that this lesson be introduced by a discussion of some common physical activities, some of which are commutative and some which are not. (See page 164 of the *Teaching Guide, Seeing Through Mathematics, Book I, Special Edition.*)

The students can then be lead through the use of a number of examples involving natural numbers to the generalization, $a + b = b + a$ and $ab = ba$.

Lesson 63 — The Associative Properties

The associative properties of addition and multiplication are properties when the sum or product of *three* numbers is to be found. The commutative properties are order properties using only *two* numbers.

It is suggested that students be lead to the generalization through the use of a number of examples, physical and numerical.

The associative property can be used to simplify computation. For example,

$$67 + 8 + 92$$

$$67 + (8 + 92)$$

$$67 + 100$$

$$167$$

Lesson 64 — The Distributive Property

The distributive property links together the operations of addition and multiplication.

This lesson becomes more meaningful to students if it is begun by illustrating the property using groupings of objects. Popisicle sticks or "pop-beads" are useful aids for such activities.

Knowledge of the distributive property can be helpful in certain computations. The presentation of applications of the kind suggested on page 168, *Teaching Guide, Seeing Through Mathematics, Book I, Special Edition,* motivates the student and reinforces his understanding of the property.

Lesson 65 — Simplifying Expressions

This lesson presents certain accepted procedures for simplifying expressions. Students should be expected to master the rules thoroughly.

Lesson 66 — The Natural Number System

This lesson brings together all the characteristics of the natural number system studies to date. It is suggested that pupils compile a list of characteristics similar to the one presented on page 282 of this textbook as a summary exercise before they begin the study of this lesson.

Lesson 67 — A Finite Number System

The purpose of this lesson is to present a review of the characteristics of number systems studied in the preceding lessons, and to show that these same characteristics apply to other number systems.

No student should be expected to memorize the definition of addivision and muldivision, or to learn the tables associated with these operations.

Lesson 68 — Zero and One

This lesson develops the identity elements for multiplication and addition. These ideas are easily grasped by students when presented in a straightforward manner. *The Teaching Guide, Seeing Through Mathematics, Book I, Special Edition,* suggests that the part of the lesson dealing with identity elements as they are developed using set ideas may be omitted if classes are not able to maintain the time schedule planned.

Lesson 69 — Composite and Prime Numbers

Notice that 1 (one) is not a prime number under the definition given in the text.

Students enjoy using the Sieve of Eratosthenes to find all the prime numbers between one and any other given number.

Note that twin primes are pairs of prime numbers that have exactly one counting number between them. The prime numbers 3, 5, 7 represent the only instance of triple primes.

Students should be expected to master the ideas in this lesson well.

Lesson 70 — The Unique Factorization Property

Students must know the distinction between writing the factors of a number and the complete factorization of a number. The set of factors of 18 is (1, 2, 3, 6, 9, 18). The complete factorization of 18 is shown as follows: $18 = 2 \cdot 3 \cdot 3$.

The approach to complete factorization used in the text is good for developing understanding; but after the understanding has developed, students find the alternate approach offered in the *Teaching Guide, Seeing Through Mathematics, Book I, Special Edition,* to be much faster.

"Least common multiple" can be presented at the same time as "greatest common factor", since both arise directly from complete factorization.

UNIT 7 — The Rational Numbers of Arithmetic

The lesson material presented in this unit is essential to each student's mathematical background, and should be taught in full detail. This is likely to require 10 to 12 weeks of teaching time.

The textbook lessons are carefully prepared and include content familiar to most teachers, even though the approach may be somewhat different from that in former mathematics programs.

For these reasons, no lesson-by-lesson suggestions have been outlined. However, a brief overview of the entire unit has been presented.

I FRACTIONS AND EQUIVALENT FRACTIONS

a. Teachers should understand that the rational numbers of arithmetic are developed as a number system with certain members, operations and properties. This unit begins with a fraction as a special kind of ordered pair of numbers representing parts of a whole. Note the different ways of writing an ordered pair: for example, (1, 4), 1/4, $\frac{1}{4}$. For convenience, the symbol $\frac{1}{4}$ is used to express a fraction.

b. Any natural number which is a member of the set $N = 0, 1, 2, \ldots$ may be used in the numerator, while any natural number except zero may be used in the denominator of a fraction. Using these sets for each replacement of the variables, if a true statement is obtained from $a \times d = c \times b$, then $\frac{a}{b}$ is equivalent to $\frac{c}{d}$. This definition is reversible, so that if $\frac{a}{b}$ is equivalent to $\frac{c}{d}$, then a true statement is obtained from $a \times d = c \times b$, using natural numbers set as a universe for the numerator and the natural numbers except zero as a universe for the denominator.

II RATIONAL NUMBERS

a. The principle of reduction is used to develop the set of the rational numbers of arithmetic. Reduction by division may be used only when it is possible to divide both the numerator and denominator of a given fraction by the same counting number without a remainder.
Students will need considerable drill in using the reduction property.

b. The definition of a rational number of arithmetic as a set whose numbers are a basic fraction and all fractions equivalent to it must be thoroughly understood by the student.

c. Lessons 74 and 77, dealing with the multiplication and division of fractions, may be combined in one presentation. The generalizations developed in these lessons must be understood by the students.

d. The operations of addition and subtraction of rational numbers as presented in Lessons 75 and 76 may be combined in one presentation. Care must be taken to drill the students in the addition and subtraction of fractions horizontally.

e. Since the reciprocal property forms a most important concept for future work, the teacher should emphasize Lesson 84.

III DECIMAL NUMBERS

a. Students should be thoroughly conversant with the place values used in the decimal system.

b. Emphasize the operations with decimals. The division algorism used in *Seeing Through Mathematics, Book I, Special Edition,* is a continuation of the method begun in the elementary grades. However, students using the common division algorism will need extra consideration by the teacher in order to make Lessons 87 and 88 meaningful.

IV PROBLEMS

In Unit 7, problems with compound conditions in one variable when the universe is the set of rational numbers of arithmetic, simple or compound conditions in one or two variables for rate problems that involve rational numbers, and problems that involve per cents and rational numbers are presented. Specific business terms are included in some problems. Students must understand the meaning of these terms if they are to be successful in doing the problems.

Date Due

FEB 8 '67			
JUL 8 '67			
JUL 2 4 '67			

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